

## **Using Vectors to Construct Carbon Nanotubes (Answer Key)**

### **Audience**

High-school chemistry, physics, and mathematics students

### **Time Frame**

Set-up: 10 minutes

Activity: 55 minutes

Clean-up: 5 minutes

### **Objectives**

After completing the activity, students will be able to:

1. identify and name the three types of carbon nanotubes; and
2. add vectors tail to head to determine the circumference of a carbon nanotube

### **National Science Education Content Standards Addressed**

Science as Inquiry

CONTENT STANDARD A: As a result of activities in grades 9-12, all students should develop

- Abilities necessary to do scientific inquiry
- Understandings about scientific inquiry

Physical Science

CONTENT STANDARD B: As a result of their activities in grades 9-12, all students should develop an understanding of

- Structure of atoms
- Structure and properties of matter
- Chemical reactions
- Motions and forces
- Conservation of energy and increase in disorder
- Interactions of energy and matter

### **Principles & Standards for School Mathematics (<http://www.nctm.org/standards/>)**

Geometry (grades 9-12 expectations): Apply transformations and use symmetry to analyze mathematical situations. In grades 9–12 all students should –

- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices

### **Activity Materials (per pair of students)**

- one overhead transparency of a graphene sheet [hexagons\_for\_transparency.doc]
- armchair and zig-zag “sleeve” [hex\_sleeve.doc]
- “blue-tak”
- overhead marker(s)

### **Background**

Many elements exist in two or more different forms. These unique chemical species are known as *allotropes*. The element carbon has many allotropes, some more common than others. The most well-known carbon allotropes are diamond and graphite. Even though they are both made of only carbon atoms, diamond and graphite have very different properties due to the way the atoms are bonded to each other.

In diamond, the hardest known material on earth, each carbon atom is bonded to four other carbon atoms (Figure 1). Each bond is equal in strength, causing diamond to look like a single giant molecule. In graphite, each carbon is strongly bonded to three carbon atoms and weakly bonded to one carbon atom (Figure 2). This means that graphite is made of layers of carbon atoms arranged in the shape of hexagons (called *graphene* sheets) that easily slide past each other. Graphite is used as a lubricant and in pencil “lead” because of the ease of graphene layers sliding past each other.

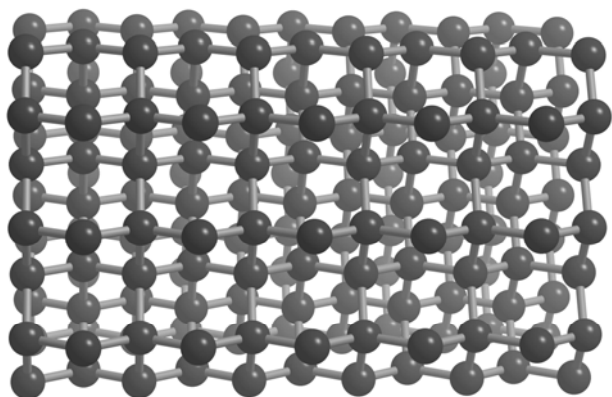


Fig. 1. Ball and stick model of diamond

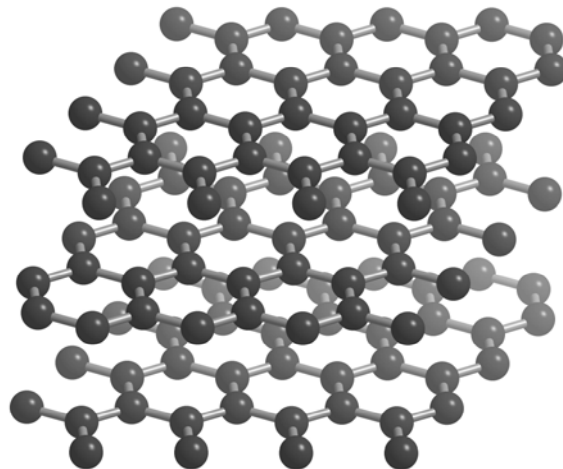


Fig. 2. Ball and stick model of graphite

A third allotrope of carbon is the *nanotube* (Figures 3 and 4). As the name implies, carbon nanotubes are very small (*nano-*) and cylindrical (*-tube*) in shape. The average diameter of a nanotube is 1.2 nm, and its length can reach up to 1 mm. Nanotubes look as if a graphene sheet has been rolled up into a cylinder, although they are actually formed as the cylinder “grows” in length.

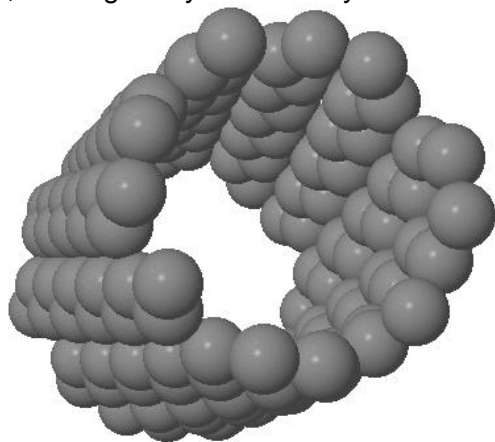


Fig. 3. Space-filling model of a carbon nanotube

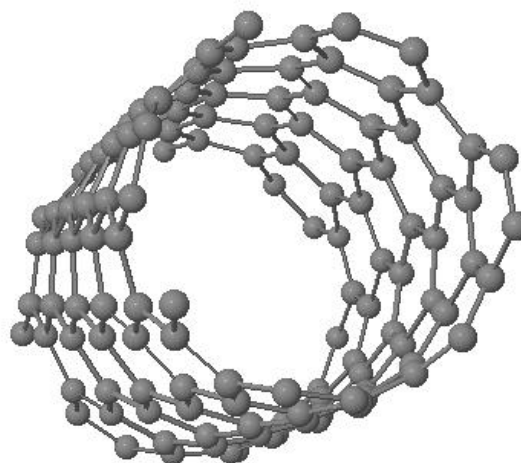


Fig. 4. Ball and stick model of the same carbon nanotube

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Nanotubes can be grouped into different categories based on the amount of twist in the pattern of the carbon atoms around the nanotube's circumference (C). The three types of nanotubes are armchair, zig-zag, and chiral (Figure 5).

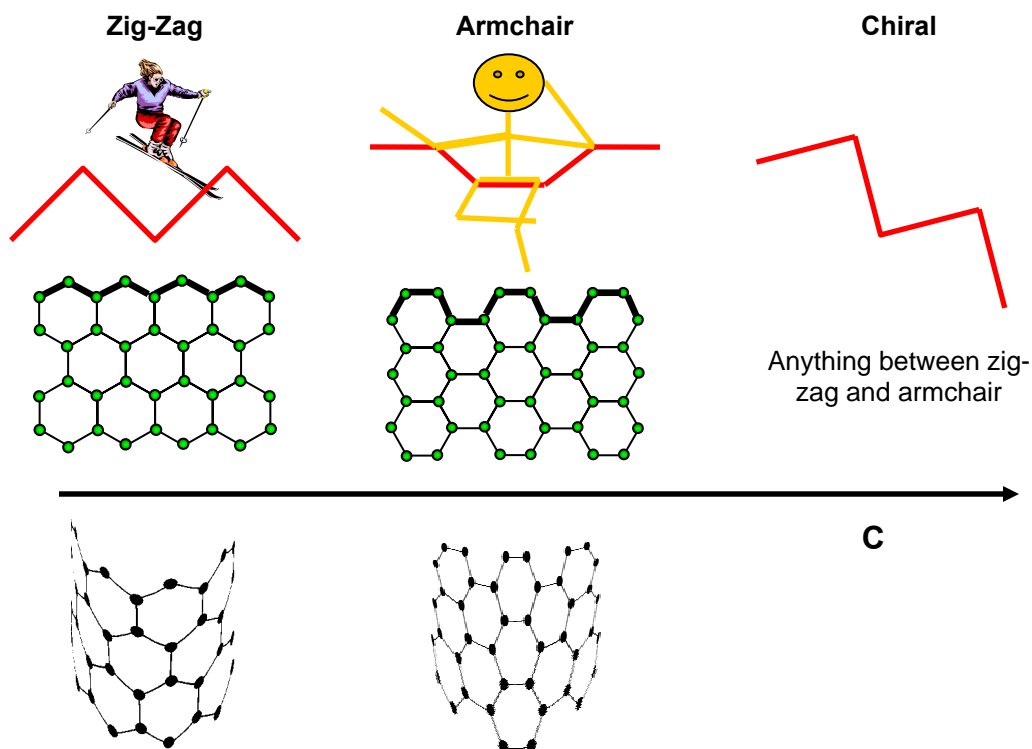


Fig. 5. The three different kinds of carbon nanotubes: zig-zag, armchair, and chiral.

**Activity Instructions**

**Set-up** (10 min)

Print enough hex\_sleeve.doc and hexagons\_for\_transparency.doc files onto clear overhead transparency sheets so that each pair of students can have a set. The overhead sheet with the hexagon sleeves needs to be cut along the dotted line either by the teacher before the activity or by the students at the beginning of the activity.

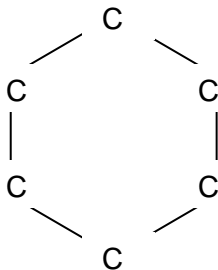
**Introduction** (10 min)

The teacher can decide if the Introduction is best conducted as a whole-class or an independent activity for the students.



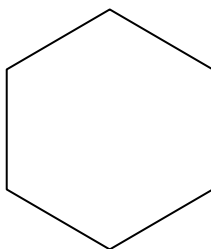
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1a) When six carbon atoms bond together, they can form a ring in the shape of a hexagon that looks like this:



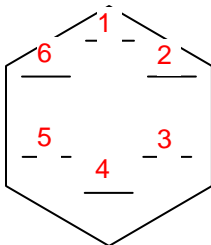
“cyclohexane”

1b) Sometimes the letter “C”s are omitted, and, instead, each vertex represents a carbon atom like this:



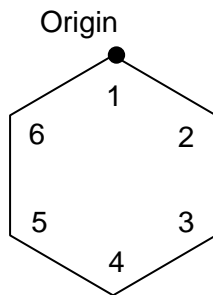
“cyclohexane”

1c) To distinguish between each carbon atom, each is assigned a number. Starting with the carbon on top, and moving in a clockwise direction, assign numbers one through six to the carbons in the hexagon below.



“cyclohexane”

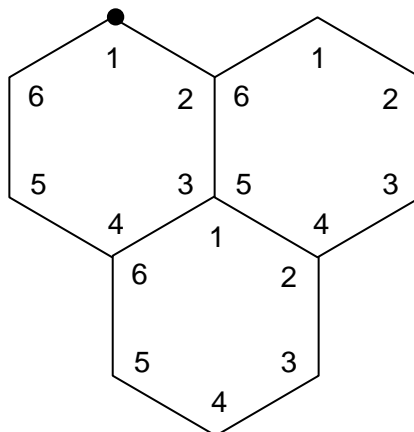
2a) Locate the origin on your graphene sheet overhead transparency and number the carbons one through six.





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2b) In the same fashion, number the carbon atoms in the hexagon directly to the right and to the bottom right of your original hexagon.



3a) All carbon atoms with the same number are called *analogous* carbons. Keeping track of analogous carbons allows us to identify the type of carbon nanotube we have. The arrows shown in the picture below (Figure 6) connect the carbon at the origin to its neighboring analogous carbon atoms. These arrows are called *vectors*. You are probably already familiar with (x,y) vectors that represent horizontal and vertical directions on a graph (Figure 7). Since carbon nanotubes have vectors that are not perpendicular to each other, they are given different names;  $a_1$  and  $a_2$  (Figure 6).

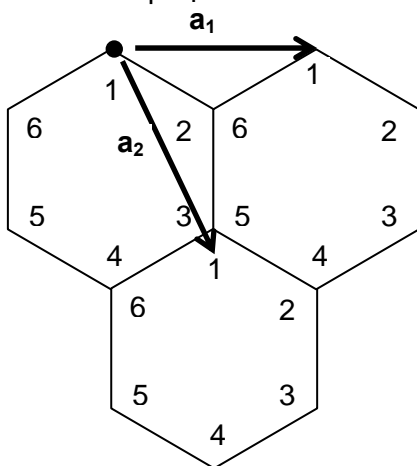


Fig. 6.

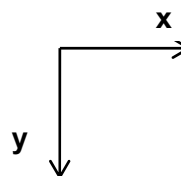
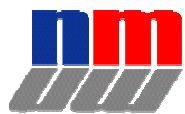


Fig. 7.

3b) To name a nanotube, you must count the number of analogous carbons it takes in each direction to get you back to your origin carbon atom. The letter  $n$  is used to indicate the number of analogous carbons in the  $a_1$  direction, and  $m$  is used to indicate the number of analogous carbons in the  $a_2$  direction. The resultant vector is represented by the set of coordinates (n,m).





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Transparency #1 (15 min)

Making a (3,0) Nanotube

4a) In a (3,0) nanotube,  $n = 3$  and  $m = 0$ . On your graphene sheet overhead transparency, start at the origin and move three analogous carbons ( $n$ ) in the  $a_1$  direction. Use a transparency marker to draw a dot at the (3,0) position (Figure 8).

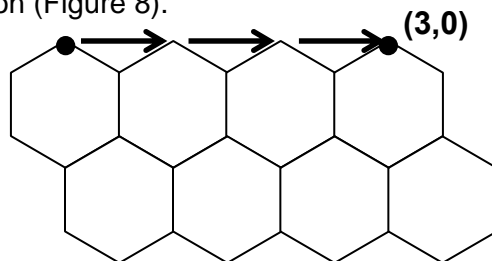


Fig. 8.

Place a small amount of blue-tak on the carbon at the origin and roll up the graphene sheet so the two dots are superimposed on each other. Gently adhere the blue-tak to the (3,0) carbon so that the tube stays rolled up. Place each sleeve around the circumference of the tube and match them up to determine what type of nanotube you have. Record this information in the data table.

4b) Vectors are added by connecting them tail to head in series. The sum of the three  $a_1$  vectors here represents the circumference of the tube. The sum of any series of vectors is drawn from the tail of the first vector to the head of the last vector. Record the circumference of this nanotube in the data table. (Answer key version of the data table given below.)

Making a (5,0) Nanotube

5) Using the same procedure as Step 4, make a (5,0) nanotube and determine the type of nanotube produced and its circumference. Record this information in your data table.





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### Transparency #2 (15 min)

#### Making a (3,3) Nanotube

6a) In a (3,3) nanotube,  $n = 3$  and  $m = 3$ . Start at the origin and move three analogous carbons ( $n$ ) in the  $a_1$  direction and then three analogous carbons ( $m$ ) in the  $a_2$  direction. Draw a dot with a transparency marker at the (3,3) position (Figure 9).

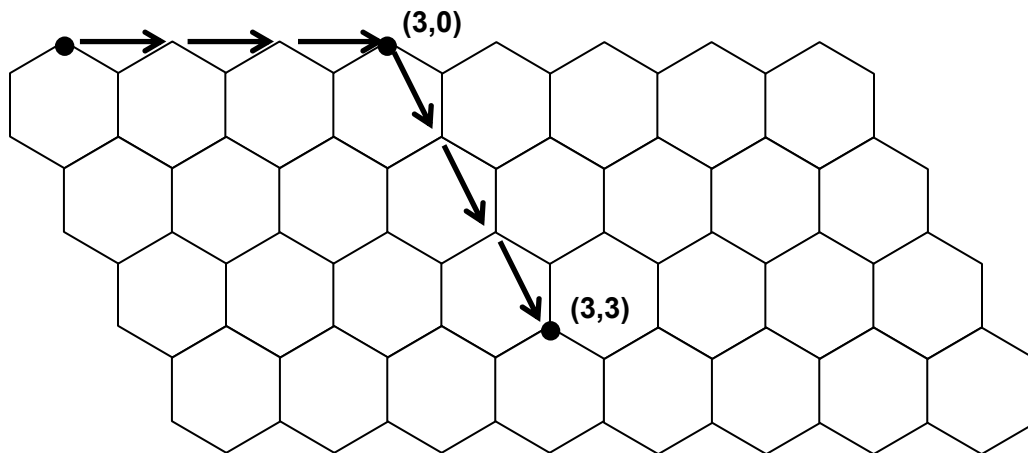


Fig. 9.

Again, place the blue-tak on the carbon at the origin and roll up the graphene sheet so the two dots are superimposed on each other. Adhere the blue-tak to the (3,3) carbon so that the tube stays rolled up. You may need to adjust the transparency so that the hexagons in each layer are lined up with each other. If necessary, additional blue-tak may be used to help hold the nanotube in place. Once again, place each sleeve around the circumference of the tube to determine the type of nanotube you have. Record this information in the data table.





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6b) Notice that these vectors are also added by connecting them tail to head in series (dotted line). To determine the circumference of the nanotube, you can count the number of hexagons on the nanotube, or resolve the vectors mathematically. Record this information in the data table.

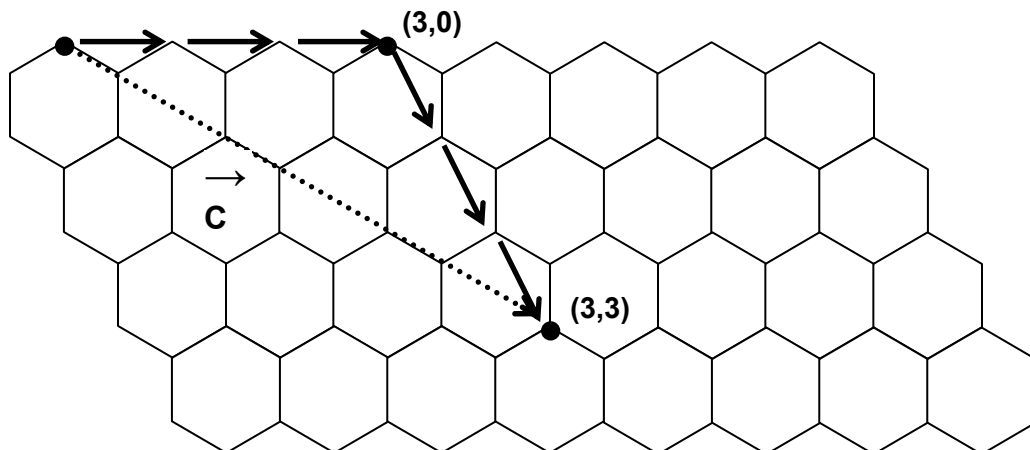


Fig. 9.

*Making a (2,2) Nanotube*

7) Using the same procedure as Step 6, make a (2,2) nanotube and determine the type of nanotube produced and its circumference. Record this information in your data table.

**Transparency #3** (10 min)

*Making a (3,2) Nanotube*

Using what you have learned in this activity, make a (3,2) nanotube and record your results in the data table.

**Data Table (Answer Key)**

Coordinates	Type of CNT	Circumference
(3,0)	zig-zag	3.00
(5,0)	zig-zag	5.00
(3,3)	armchair	5.20
(2,2)	armchair	3.46
(3,2)	chiral	X

**Conclusion** (5 min)

After the activity, the teacher should conclude the activity by reviewing the three types of carbon nanotubes (zig-zag, armchair, and chiral), how to identify each type, and the process of adding vectors tail to head to obtain the circumference of the nanotube. Check the students' data tables to assess their understand of using vectors to identify carbon nanotubes.





# Interdisciplinary Education Group

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### **Background**

For background, see O.M. Castellini, G.C. Lisenksy, J. Ehrlich, G.M. Zenner, and W.C. Crone, "The Structures and Properties of Carbon," *The Science Teacher*, p. 36-41, December 2006.

Students also need to understand vectors in order to do this activity. A science and a math teacher could team up to work with students on the project, or, alternatively, students could review vectors on-line. The following web-pages are possibilities: "Vector Addition Tutorial," <http://www.1728.com/vectutor.htm> (accessed December 13, 2006); OR "The Physics Classroom: Lesson 1: Vectors," <http://www.glenbrook.k12.il.us/GBSSCI/PHYS/Class/vectors/u3l1a.html> (accessed December 13, 2006).

### **Supplemental Materials**

- Armchair and zig-zag "sleeve" – [hex\_sleeve.doc]
- Overhead transparency of a graphene sheet – [hexagons\_for\_transparency.doc]
- "Vector Addition Tutorial," <http://www.1728.com/vectutor.htm> (accessed December 13, 2006)
- "The Physics Classroom: Lesson 1: Vectors," <http://www.glenbrook.k12.il.us/GBSSCI/PHYS/Class/vectors/u3l1a.html> (accessed December 13, 2006)

### **References**

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